The wave drag at zero lift of slender delta wings and similar configurations

By M. J. LIGHTHILL
Department of Mathematics, University of Manchester

(*Received* 29 *March* 1956)

SUMMARY

Ward's slender-body theory of supersonic flow is applied to bodies terminating in either (i) a single trailing edge at right angles to the oncoming supersonic stream, or (ii) two trailing edges at right angles to one another as well as to the oncoming stream, or (iii) a cylindrical section with two or four identical fins equally spaced round it. The wave drag at zero lift, D, is given by the expression

$$\frac{D}{\frac{1}{2}\rho U^2} = \frac{1}{2\pi}\int_0^l\int_0^l \log\frac{l}{|s-z|}\, S''(s)S''(z)\, ds dz - \\ - \frac{S'(l)}{\pi}\int_0^l \log\frac{l}{l-z}\, S''(z)\, dz + \frac{S'^2(l)}{2\pi}\left\{\log\frac{l}{(M^2-1)^{1/2}b} + k\right\},$$

where l is the length of the body, b the semi-span of the trailing edge (or length of trailing edge of a single fin), and $S(z)$ is the cross-sectional area of the body at a distance z behind the apex. The constant k depends on the distribution of trailing-edge angle along the span for each trailing-edge configuration. In case (i) it is 1·5 for a uniform distribution of trailing-edge angle and 1·64 for an elliptic distribution. In case (ii) it is 1·28 for a uniform distribution and 1·44 for an elliptic distribution. Study of case (iii) indicates that interference effects due to the presence of the body reduce the drag of the fins. For example, with a uniform distribution of trailing-edge angle, k for two fins falls from 1·5 in the absence of a body to 1·06 when the body radius equals the trailing-edge semi-span, while k for four fins falls from 1·28 to 0·45 under the same conditions.

Where ordinary finite-wing theory is applicable, the present method must agree with it for small $(M^2-1)^{1/2}b/l$, and this is confirmed by two examples (§ 3), but within the limit imposed by slenderness the present method is of course more widely applicable, as well as simpler, than finite-wing theory.

It is not known experimentally whether slender-body theory gives accurate predictions of drag at zero lift, for the shapes here discussed, under the conditions for which on theoretical grounds it might be expected to do so. It should be noted that, although tests have not yet been made on ideally suitable bodies, no clear

indication of the variation with $\log(M^2-1)$ indicated by the formula has ever been found.

1. Introduction

Ward's 'slender-body' theory (1949) applies to supersonic flow about 'bodies' (here understood to include wings and wing-body combinations) whose surface is headed by a pointed apex and makes everywhere an angle with the undisturbed stream small compared with the Mach angle. Then the right-hand side in the linearized equation of motion for ϕ, the disturbance potential*,

$$\frac{\partial^2\phi}{\partial r^2}+\frac{1}{r}\frac{\partial\phi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2}=(M^2-1)\frac{\partial^2\phi}{\partial z^2} \tag{1}$$

(in cylindrical coordinates with the z-axis parallel to the undisturbed stream), may be expected to be small near the body compared with the left-hand side, provided that the shape of the body varies smoothly with z.

Accordingly, ϕ may be determined in this region as a solution of Laplace's equation, in a plane perpendicular to the undisturbed stream, which satisfies the boundary condition at the body surface. The solution that satisfies

$$\phi=\frac{S'(z)}{2\pi}\log r+\frac{1}{2\pi}\int_0^z \log\frac{(M^2-1)^{1/2}}{2(z-s)}S''(s)\,ds+O\left(\frac{1}{r}\right) \tag{2}$$

as $r\to\infty$ must be selected if it is to join smoothly, far from the body, on to a solution of (1) which is zero upstream of the Mach cone from the apex.

This paper is concerned with bodies which terminate in a straight trailing edge at right angles to the stream, with a small but finite trailing-edge angle (which may vary along it). This class of bodies includes delta wings. Bodies terminating in two such trailing edges at right angles to one another, or in a cylindrical section with two or four fins attached, are also treated.

Ward and others have used his theory to obtain the lift, moment and drag due to lift for such bodies. In this paper their drag at zero lift is investigated. The Ward theory is applied without change, so it may be asked why a separate paper is necessary. This is partly because Ward has suggested that the theory is inapplicable to cross-sections with large curvature, and the trailing-edge cross-sections described above have infinite curvature at the tips One might hope, however, that this suggestion of Ward's would be correct only if fluid flows round the region of high curvature (leading to specially low pressures there), and that in cases like those considered here, where this is not so, the theory can still be alppied.

Another reason is that many people misinterpret Ward's theory and suggest that it equates drag at zero lift to that of a body of revolution with the same distribution $S(z)$ of cross-sectional area with distance from the apex. For the bodies here considered, this 'equivalent body of revolution'

* Such that $U(z+\phi)$ is the velocity potential, where U is the velocity of the undisturbed stream.

would have a rounded rear end, which might lead one to suppose the theory inapplicable, especially as the formal expression for drag calculated for this body shape would be infinite.

However, Ward's actual drag formula for a body of length l is

$$\frac{D}{\frac{1}{2}\rho U^2} = \frac{1}{2\pi}\int_0^l\int_0^l \log\frac{l}{|s-z|}\,S''(s)S''(z)\,ds dz - \\ - \frac{S'(l)}{2\pi}\int_0^l \log\frac{l}{l-z}\,S''(z)\,dz - \left(\oint \phi\frac{\partial\phi}{\partial\nu}\,d\tau\right)_{z=l} \quad (3)$$

(Ward 1949, equation (37); Ward 1955, equation (9.8.7); here, $d\nu$ and $d\tau$ are line elements in the plane of a cross-section, normal and tangential to it respectively; the 'base pressure' term has been omitted). Now, the first two terms in (3) are independent of the shapes of cross-sections, but the last term, an integral round the rear cross-section, is not. It is because this term vanishes in so many cases at zero lift that the theory of the equivalent body of revolution has been assumed to be true more generally than it really is.

This third term is calculated below for bodies terminating in a straight trailing edge, leading to the drag formula*

$$\frac{D}{\frac{1}{2}\rho U^2} = \frac{1}{2\pi}\int_0^l\int_0^l \log\frac{l}{|s-z|}\,S''(s)S''(z)\,ds dz - \\ - \frac{S'(l)}{\pi}\int_0^l \log\frac{l}{l-z}\,S''(z)\,dz + \frac{S'^2(l)}{2\pi}\left\{\log\frac{l}{(M^2-1)^{1/2}b} + k\right\}, \quad (4)$$

where $2b$ is the length of the trailing edge and the constant k is given by the equation

$$k = \int_{-b}^{b}\int_{-b}^{b} \log\frac{2b}{|y-Y|}\,\epsilon(y)\epsilon(Y)\,dy dY \bigg/ \left(\int_{-b}^{b}\epsilon(y)\,dy\right)^2, \quad (5)$$

where $\epsilon(y)$ is the trailing-edge angle at a distance y from the middle. For a uniform distribution of trailing-edge angle $k = 1{\cdot}5$, while for an elliptic distribution $k = 1{\cdot}64$.

Equation (4) is directly applicable to delta wings when the parameter $(M^2-1)^{1/2}b/l$ is small. Experimental work by Love (1949) and others indicates that this is perhaps the only region in which any linear theory is adequate for predicting the wave drag at zero lift†, although some results of Herbert (1951) discussed in § 3, where a particular wing shape introduced by Squire (1951) is considered as an example, are a reminder that it should not be applied with M too near unity.

In § 4 a similar analysis is applied to other trailing-edge configurations, leading always to the same form (4) for the drag with different expressions for k. The extent to which a 'dart' formed of two slender delta wings at right angles (with a common line of symmetry) has a drag approximately equal to four times that of each separately, as suggested by the theory of the

* The replacement of the 2π in the second term in (3) by π in (4) is *not* a misprint.

† Perhaps because outside this region either doubtful assumptions about 'leading-edge forces' have to be made, or else the flow normal to some edge is nearly sonic.

equivalent body of revolution, is discussed; the true slender-body theory drag is somewhat less than this.

An analysis (§ 4) of bodies terminating in cylindrical sections with slender fins having trailing edges at right angles to the stream shows that interference effects due to the presence of the body tend to reduce the drag of the fins. It had been widely supposed that this would not be true in the case of two fins, but the present result, which is subject to the usual limitations on accuracy of slender-body theory but can hardly be without qualitative validity, indicates that caution is necessary in inferring the drag of delta wings from flight tests on cylindrical projectiles which carry them in the form of fins.

2. General theory for a straight trailing edge perpendicular to the undisturbed stream

In the plane, at right angles to the stream, which includes the trailing edge, we choose Cartesian coordinates (x, y) such that the trailing edge is $x = 0$, $-b < y < b$. The boundary condition on ϕ in this plane is

$$\frac{\partial\phi}{\partial x} = \begin{Bmatrix} -\frac{1}{2}\epsilon(y) & (x = +0) \\ +\frac{1}{2}\epsilon(y) & (x = -0) \end{Bmatrix} \quad (-b < y < b), \tag{6}$$

where $\epsilon(y)$ is the trailing edge angle at a distance y from the centre. A solution of Laplace's equation in the (x,y) plane satisfying this boundary condition is obtained by a sink distribution, the sink strength in the interval dy being $\epsilon(y)\,dy$; this gives

$$\phi(x,y) = -\int_{-b}^{b} \frac{\epsilon(Y)}{2\pi} \log\{x^2+(y-Y)^2\}^{1/2}\,dY + C, \tag{7}$$

where C is an arbitrary constant. Then as $r = (x^2+y^2)^{1/2} \to \infty$,

$$\phi + \left(\int_{-b}^{b} \frac{\epsilon(Y)}{2\pi}\,dY\right)\log r = \phi - \frac{S'(l)}{2\pi}\log r \to C, \tag{8}$$

since evidently $S'(l) = -\int_{-b}^{b} \epsilon(Y)\,dY$; and hence, by (2),

$$C = \frac{1}{2\pi}\int_0^l \log\frac{(M^2-1)^{1/2}}{2(l-z)}\, S''(z)\,dz. \tag{9}$$

Now, Ward's integral, which has to be taken around the boundary of the rear cross-section, is

$$\begin{aligned} \oint \phi\frac{\partial\phi}{\partial\nu}\,d\tau &= \int_{-b}^{b}\left(\phi\frac{\partial\phi}{\partial x}\right)_{x=+0} dy + \int_{-b}^{b}\left(-\phi\frac{\partial\phi}{\partial x}\right)_{x=-0} dy \\ &= -\int_{-b}^{b} \epsilon(y)(\phi)_{x=0}\,dy \\ &= CS'(l) + \frac{1}{2\pi}\int_{-b}^{b}\int_{-b}^{b} \epsilon(y)\epsilon(Y)\log|y-Y|\,dy\,dY, \end{aligned} \tag{10}$$

by (7). A combination of equations (3), (9) and (10) gives at once the drag formula (4), with the expression (5) for the constant k. This expression for k has the merit that it is independent of b or of the scale of $\epsilon(y)$, depending only on the distribution of trailing-edge angle along the span.

Thus, for a uniform distribution ($\epsilon(y)$ = constant),

$$k = \frac{1}{4}\int_{-1}^{1}\int_{-1}^{1}\log\frac{2}{|y-Y|}\,dy\,dY = \tfrac{3}{2} = 1{\cdot}5, \tag{11}$$

while, for an elliptic distribution,

$$k = \frac{4}{\pi^2}\int_{-1}^{1}\int_{-1}^{1}(1-y^2)^{1/2}(1-Y^2)^{1/2}\log\frac{2}{|y-Y|}\,dy\,dY$$
$$= \log 4 + \tfrac{1}{4} = 1{\cdot}636. \tag{12}$$

3. Examples: application to delta wings described by Squire and by Puckett

Squire (1951) considers a delta wing all of whose cross-sections by planes normal to the stream are ellipses. The major semi-axis is bz/l and the minor semi-axis is $2\tau z(1-z/l)$, where τ is the thickness-chord ratio of the central section of the wing*. Slender-body theory is applicable to such wings if $(M^2-1)^{1/2}b/l$ is small.

The cross-sectional area distribution for this wing is

$$S(z) = \pi(bz/l)\{2\tau z(1-z/l)\}. \tag{13}$$

The trailing-edge angle is equal to 4τ at the centre and varies elliptically over the length $2b$ of the trailing edge. Note also that

$$S'(l) = -2\pi\tau b, \quad S''(z) = 4\pi\tau\frac{b}{l}\left(1-\frac{3z}{l}\right). \tag{14}$$

Substitution of these values in (5) gives for the three terms (respectively)

$$\frac{D}{\frac{1}{2}\rho U^2} = \tfrac{15}{2}\pi\tau^2b^2 - 10\pi\tau^2b^2 + 2\pi\tau^2b^2\left\{\log\frac{l}{(M^2-1)^{1/2}b} + 1{\cdot}636\right\}, \tag{15}$$

the value of k appropriate to an elliptic distribution of trailing-edge angle being inserted. The drag coefficient based on the area lb of the wing planform is therefore

$$C_D = \frac{D}{\frac{1}{2}\rho U^2 lb} = 2\pi\tau^2\frac{b}{l}\left\{\log\frac{l}{(M^2-1)^{1/2}b} + 0{\cdot}386\right\}. \tag{16}$$

This agrees with the asymptotic form of Squire's expression for C_D for very small values of $(M^2-1)^{1/2}b/l$, but is somewhat smaller for moderate values, as figure 1 shows. (Note that the form of Squire's expression depends on the precise hypothesis used to get over the notorious 'leading-edge force' difficulty, but that the asymptotic form for small $(M^2-1)^{1/2}b/l$ is independent of this, as no large pressure coefficient at the leading edge arises under this condition.)

* This central section is biconvex, but there is a round-nosed aerofoil section at all other spanwise positions.

The free-flight measurements of Herbert (1951), in the range

$$(M^2-1)^{1/2}b/l < 0{\cdot}4$$

where the slender-body theory can reasonably be applied, are well below the theoretical values, but the wing used by him was so far from slender (in fact $b/l = 1$) that the condition mentioned holds only when $M < 1{\cdot}08$, so that the problem has become a transonic one. It is hoped that a transonic version of slender-body theory may one day be developed*, but this has not yet been done.

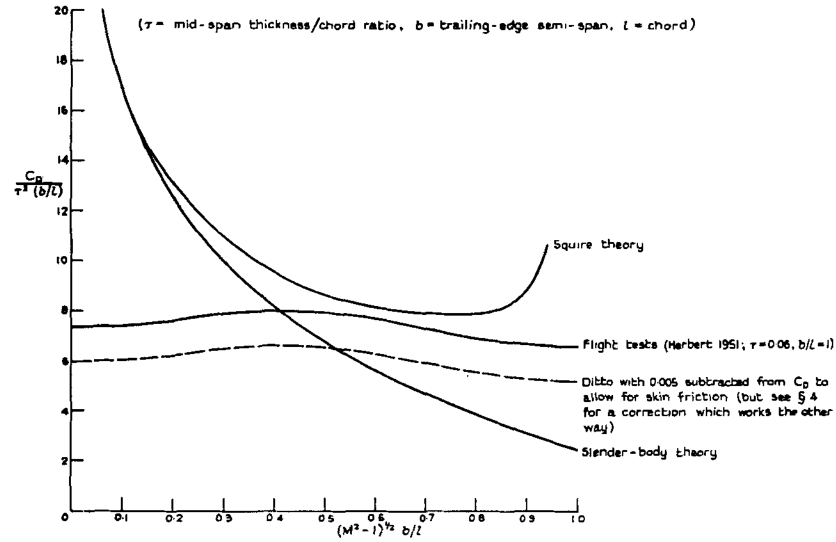

Figure 1. Drag results for Squire wing.

As another example, consider a delta wing with a double-wedge aerofoil section of uniform thickness-chord ratio τ (Puckett 1946). Then the cross-sectional area distribution is

$$S(z) = \begin{cases} 2\tau b z^2/l & (z < \tfrac{1}{2}l), \\ 2\tau b(4z-l-3z^2/l) & (z > \tfrac{1}{2}l). \end{cases} \tag{17}$$

The trailing-edge angle is 2τ all along the span, and

$$S'(l) = -4\tau b;\; S''(z) = \begin{cases} 4\tau b/l & (z < \tfrac{1}{2}l), \\ -12\tau b/l & (z > \tfrac{1}{2}l). \end{cases} \tag{18}$$

Substitution in (5) gives for the three terms (respectively)

$$\frac{D}{\frac{1}{2}\rho U^2} = \frac{8\tau^2 b^2}{\pi}\left(\tfrac{3}{2} + 4\log 2\right) - \frac{8\tau^2 b^2}{\pi}(2+4\log 2) + + \frac{8\tau^2 b^2}{\pi}\left\{\log\frac{l}{(M^2-1)^{1/2}b} + \tfrac{3}{2}\right\}, \tag{19}$$

* This would require the modification of the second term in (2), to allow for the fact that equation (1), on to a solution of which the harmonic function ϕ must join far from the body, is modified by addition of the ' transonic ' term

$$(\gamma+1)M^2\frac{\partial\phi}{\partial z}\frac{\partial^2\phi}{\partial z^2}$$

to its right-hand side.

the value of k for a uniform distribution of trailing-edge angle being inserted. The drag coefficient is therefore

$$C_D = \frac{8\tau^2}{\pi}\frac{b}{l}\left\{\log\frac{l}{(M^2-1)^{1/2}b}+1\right\}, \tag{20}$$

in agreement with the asymptotic form of Puckett's expression for C_D for very small values of $(M^2-1)^{1/2}b/l$. The difference is a term

$$O[(M^2-1)b^2/l^2]$$

inside the curly brackets in (20), and Ward's arguments indicate that this will generally be the case. As with the Squire wing, Puckett's expression

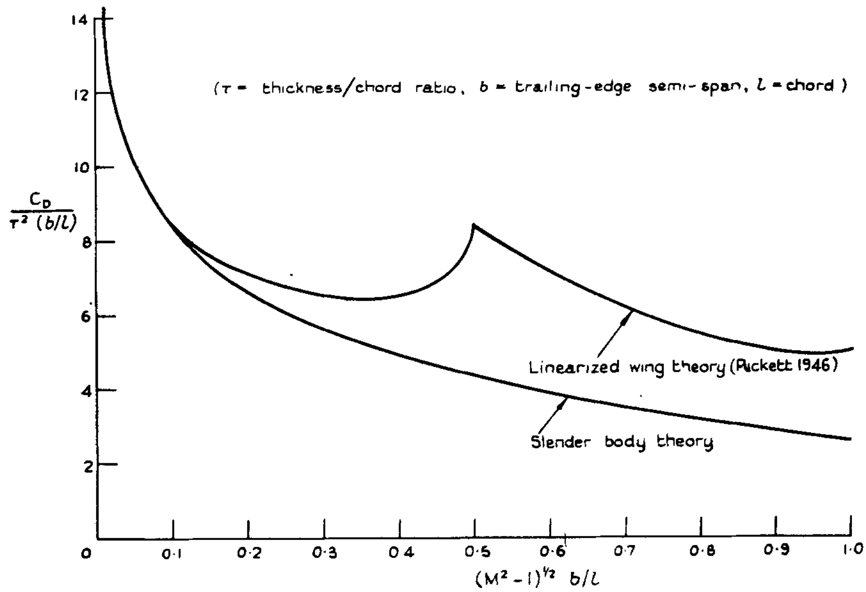

Figure 2. Drag results for delta wing with double-wedge section.

for the wave drag rises above expression (20) as the condition of slenderness is removed, as figure 2 shows.

4. Theory for other trailing-edge configurations

One interesting configuration is a body terminating in two trailing edges at right angles to one another as well as to the oncoming stream; for example, a projectile of revolution tapering to zero radius at the rear end and stabilized by cruciform fins. This can be treated by methods similar to those of § 2.

We suppose that the distribution of trailing-edge angle is the same on each trailing edge, and choose the x-axis and y-axis to lie along the two edges. Then there are two boundary conditions on ϕ, one being specified by equation (6), and the other being

$$\frac{\partial\phi}{\partial y} = \begin{cases} -\tfrac{1}{2}\epsilon(x) & (y=+0) \\ +\tfrac{1}{2}\epsilon(x) & (y=-0) \end{cases} \quad (-b<x<b). \tag{21}$$

A solution is obtained by a sink distribution as in § 2, giving

$$\phi(x,y) = -\int_{-b}^{b} \frac{\epsilon(Y)}{2\pi} \log\{x^2+(y-Y)^2\}^{1/2} dY - \\ -\int_{-b}^{b} \frac{\epsilon(Y)}{2\pi} \log\{(x-X^2)+y^2\}^{1/2} dX + C, \quad (22)$$

with C given by equation (9) as before. Note that now

$$S'(l) = -2\int_{-b}^{b} \epsilon(Y)dY.$$

Hence Ward's integral is

$$\oint \phi \frac{\partial \phi}{\partial \nu} d\tau = -\int_{-b}^{b} \epsilon(y)(\phi)_{x=0} dy - \int_{-b}^{b} \epsilon(x)(\phi)_{y=0} dx$$
$$= CS'(l) + \frac{1}{\pi}\int_{-b}^{b}\int_{-b}^{b} \epsilon(y)\epsilon(Y)\{\log|y-Y| + \log(y^2+Y^2)^{1/2}\} dydY, \quad (23)$$

where four double integrals have been combined into one by taking the variables of integration as y and Y in each. A combination of equations (3), (9) and (23) now gives the drag formula (4) with the new expression

$$k = \int_{-b}^{b}\int_{-b}^{b} \epsilon(y)\epsilon(Y) \log \frac{2b}{|y-Y|^{1/2}(y^2+Y^2)^{1/4}} dydY \Big/ \left(\int_{-b}^{b} \epsilon(y)\, dy\right)^2 \quad (24)$$

for k. As before, k depends only on the distribution of trailing-edge angle, not on its magnitude or on b. It tends to be slightly less than its previous value (5) because on the average $(y^2+Y^2)^{1/2}$ exceeds $|y-Y|$.

Thus, for a uniform distribution of trailing-edge angle,

$$k = \frac{1}{4}\int_{-1}^{1}\int_{-1}^{1} \log \frac{2}{|y-Y|^{1/2}(y^2+Y^2)^{1/4}} dydY$$
$$= \tfrac{1}{4}\log 2 + \tfrac{3}{2} - \tfrac{1}{8}\pi = 1{\cdot}281 \quad (25)$$

(compare 1·5 in the case of a single trailing edge), while for an elliptic distribution

$$k = \frac{4}{\pi^2}\int_{-1}^{1}\int_{-1}^{1} (1-y^2)^{1/2}(1-Y^2)^{1/2} \log \frac{2}{|y-Y|^{1/2}(y^2+Y^2)^{1/4}} dydY$$
$$= \log 4 + \frac{1}{2} - \frac{1}{2\pi} + \frac{1}{\pi}\int_0^1 \frac{\tan^{-1} t}{t} dt = 1{\cdot}436 \quad (26)$$

(compare 1·636 in the case of a single trailing edge).

The fact that k is less than the corresponding value for a single trailing edge means that the drag of a pair of delta wings superimposed with their apexes and axes of symmetry coinciding, and their planes at right angles to one another, is not quite four times that of each separately. For example, a 'dart' of this kind made of two Squire wings has

$$C_D = 4\pi\tau^2 \frac{b}{l}\left\{\log \frac{l}{(M^2-1)^{1/2}b} + 0{\cdot}186\right\}, \quad (27)$$

where the drag coefficient is based on the sum of the wing planform areas, and the constant is different from that in (16) owing to the different value of

k which is appropriate. However, for very slender wings the logarithmic term is the most important in (16) and (27), and then the interference between the wings almost doubles the drag of each.

We may consider also bodies which terminate in a cylindrical cross-section and a pair, or two pairs at right angles, of fin trailing edges. Let R be the radius of the cylindrical cross-section and b the length of each trailing edge, and suppose that the trailing-edge angle is $\epsilon(q)$ at a distance $q = r-R$ from the cylinder. Then a solution for ϕ in the plane, at right angles to the stream, which includes the trailing-edges, is obtainable by a sink distribution as before, but image sources and sinks in the cylinder have to be included as well, giving for the case of two fins placed opposite to one another

$$\phi(x,y) = -\int_0^b \frac{\epsilon(Q)}{2\pi}\left[\log\frac{[x^2+\{y-(R+Q)\}^2]^{1/2}[x^2+\{y-R^2(R+Q)^{-1}\}^2]^{1/2}}{(x^2+y^2)^{1/2}}+\right.$$
$$\left.+\log\frac{[x^2+\{y+R+Q\}^2]^{1/2}[x^2+\{y+R^2(R+Q)^{-1}\}^2]^{1/2}}{(x^2+y^2)^{1/2}}\right]dQ+C, \quad (28)$$

where C is given by equation (9) as before. Hence Ward's integral is

$$\oint \phi\frac{\partial\phi}{\partial\nu}\,d\tau = -\int_0^b \epsilon(q)\phi(0,R+q)\,dq-\int_0^b \epsilon(q)\phi(0,-R-q)\,dq$$
$$= CS'(l)+\frac{1}{\pi}\int_0^b\int_0^b \epsilon(q)\epsilon(Q)\times$$
$$\times\log\frac{|q-Q|\{R+q-R^2(R+Q)^{-1}\}\{2R+q+Q\}\{R+q+R^2(R+Q)^{-1}\}}{(R+q)^2}\,dqdQ. \quad (29)$$

A combination of equations (3), (9) and (29) now gives the drag formula (4) with the new expression

$$k = \int_0^b\int_0^b \epsilon(q)\epsilon(Q)\log\frac{2b}{|(R+q)^2-(R+Q)^2|^{1/2}\{1-R^4(R+q)^{-2}(R+Q)^{-2}\}^{1/2}}\,dqdQ \Big/ \left(\int_0^b \epsilon(q)dq\right)^2 \quad (30)$$

for k. This expression depends as before on the way in which the trailing-edge angle is distributed, as well as on the ratio $R/b = \alpha$, say.

For a uniform distribution of trailing-edge angle,

$$k = \int_0^1\int_0^1 \log\frac{2}{|(\alpha+q)^2-(\alpha+Q)^2|^{1/2}\{1-\alpha^4(\alpha+q)^{-2}(\alpha+Q)^{-2}\}^{1/2}}\,dqdQ, \quad (31)$$

and this integral is straightforward to evaluate if the logarithm of the curly bracket is expanded in series. This gives values of k as a function of $\alpha = R/b$ as in table 1 (see also figure 3). It is seen that the effect of interference between fins and body is to decrease the drag of the fins by decreasing k. In the limit of very large R/b (which is of more theoretical than practical interest) k becomes $\frac{3}{2}-\log 2 = 0{\cdot}807$. This may be thought surprising on the grounds that the drag of a wing composed of only the two fins (the case $R/b = 0$) ought to be reproduced for very large R/b, when each fin, being accompanied by an image in the nearby plane boundary of the cylinder, should have half the drag of such a complete wing. However, this argument neglects the effect on a fin of the image sources at the centre of the cylinder, and of the sinks (and image sinks) representing the other fin. It is the fact that the latter are twice as distant as the former, though of the same strength, which is responsible for the limiting result

$$(k)_{R/b=\infty} = (k)_{R/b=0} - \log 2.$$

Comparable behaviour is to be expected for other distributions of trailing-edge angle, and the limiting result just quoted is easily seen to be quite general.

R/b	0·0	0·1	0·25	0·5	1·0	1·5	2·0	3·0
k	1·500	1·391	1·283	1·173	1·057	0·997	0·960	0·918
$R/b>3$	$k \doteqdot 0{\cdot}807 + \dfrac{0{\cdot}398}{R/b} - \dfrac{0{\cdot}236}{(R/b)^2}$							

Table 1

Note that the limiting process $R/b \to \infty$ should be thought of as achieved by reducing b for given cylinder radius R and fin length. If instead R is increased indefinitely, keeping b and the fin length fixed, the two fins must cease to interfere with one another after a time. At the same time, slender-body theory will clearly have ceased to be applicable. Thus, there are certainly circumstances in which the interference here discussed does not take place. However, in the many practical cases when the two fins do interfere, the present theory, although only approximate, makes it appear likely that interference will at least lower the drag.

Consider a numerical example. Herbert (1951), in the experiments quoted in figure 1, attached his delta wing in the form of two fins to a body with $R/b = 0{\cdot}16$. This, by table 1, should reduce k by 0·15, and hence reduce the drag coefficient C_D (for a Squire wing with $\tau = 0{\cdot}06$ and $b/l = 1$) by $2\pi(0{\cdot}06)^2(0{\cdot}15) = 0{\cdot}0033$. This reduction is insufficient to remove the discrepancy noted in figure 1 (and there ascribed to transonic flow conditions), but such a relatively big change for such a moderate value of R/b indicates that the effect here discussed is by no means negligible.

Passing to the case of a body with cruciform fins, we easily obtain by similar means the formula

$$k = \int_0^b \int_0^b \epsilon(q)\epsilon(Q) \log \frac{2b}{|(R+q)^4-(R+Q)^4|^{1/4}\{1-R^8(R+q)^{-4}(R+Q)^{-4}\}^{1/4}}\, dq dQ \Big/ \left(\int_0^b \epsilon(q)\, dq\right)^2. \quad (32)$$

For a uniform distribution of trailing-edge angle this is

$$k = \int_0^1 \int_0^1 \log \frac{2}{|(\alpha+q)^4-(\alpha+Q)^4|^{1/4}\{1-\alpha^8(\alpha+q)^{-4}(\alpha+Q)^{-4}\}^{1/4}}\, dq dQ, \quad (33)$$

where $\alpha = R/b$ as before. This expression gives values of k as function of R/b, as in table 2 (see also figure 3).

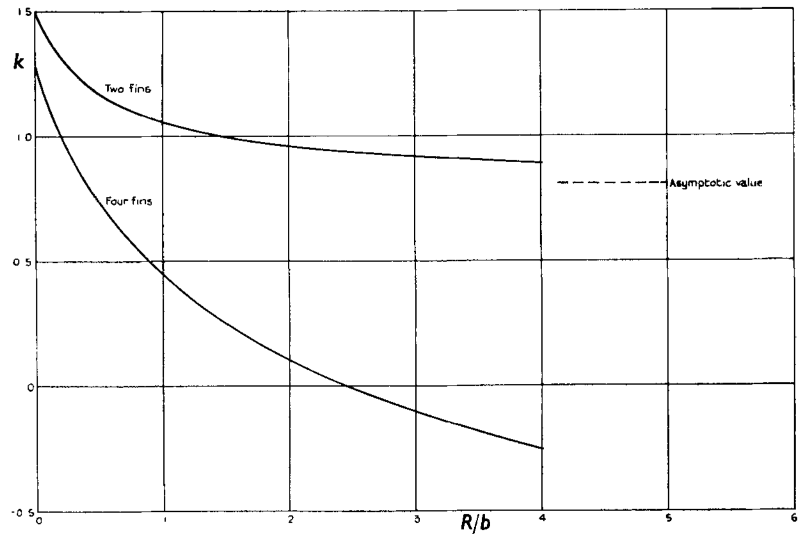

Figure 3. Values of the constant k in the expression for wave drag at zero lift given be slender-body theory, for a body terminating in a cylindrical portion of radius R, and two or four fins, the length of whose trailing edge is b in each case. The trailing-edge angle is taken as uniform.

Now, the behaviour of k exhibited in table 2 is crucially different from that in Table 1, for k decreases indefinitely as R/b is increased. This was to be expected, on the grounds that with four fins conditions for very large R/b become fundamentally different from those for small R/b. Then, quite apart from the remoter interference effects, each fin by virtue of its image in the body would have half the drag of a complete wing made up of two fins. The total drag would therefore be expected to be about twice the drag of such a wing, whereas for $R/b = 0$ slender-body theory predicts (as shown earlier) a drag only slightly less than four times the drag of a single wing.

R/b	0·0	0·1	0·25	0·5	1·0	1·5	2·0	3·0
k	1·281	1·126	0·955	0·735	0·445	0·250	0·105	−0·105

$R/b > 3$ $\qquad k \doteqdot -\tfrac{1}{2}\log\dfrac{R}{b} + 0{\cdot}403 + \dfrac{0{\cdot}199}{R/b} - \dfrac{0{\cdot}285}{(R/b)^2}$

Table 2

Hence, the steady decrease of k as R/b increases, shown in table 2 and figure 3, represents as far as slender-body theory is capable the gradual tendency to a state in which the fins cease to interfere with one another and the drag is therefore twice that of a wing made up of two of them. The final stages of the process cannot be represented by slender-body theory, but the initial trend may well be indicated fairly accurately.

References

HERBERT, P. J. 1951 Free flight measurements at supersonic speeds of the drag of a particular delta wing having a simple pressure distribution. *Aero. Res. Counc., Lond., Curr. Pap.* no. 57.

LOVE, E. S. 1949 Investigations at supersonic speeds of 22 triangular wings representing two airfoil sections for each of 11 apex angles. *Nat. Adv. Comm. Aero., Wash., Res. Mem.* no. L9D07.

PUCKETT, A. E. 1946 Supersonic wave drag of thin airfoils. *J. Aero. Sci.* **13**, 475.

SQUIRE, H. B. 1951 An example in wing theory at supersonic speed. *Aero. Res. Counc., Lond., Rep. & Mem.* no. 2549.

WARD, G. N. 1949 Supersonic flow past slender pointed bodies. *Quart. J. Mech. Appl. Math.* **2**, 75.

WARD, G. N. 1955 *Linearized Theory of Steady High-speed Flow.* Cambridge University Press.